# Classical Mechanics (20%)

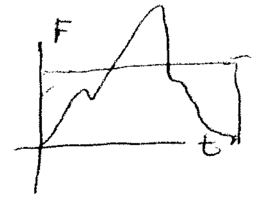
Dan Elton

$$\boxed{W = \oint_{x_1}^{x_2} F(x)\,dx}$$

$$\boxed{\vec{F} = -\vec{\nabla} V}$$

## Impulse

$$\boxed{\vec{J} = \int_{t_1}^{t_2} \vec{F}\,dt = \bar{F}\Delta t = \Delta \vec{p}}$$




$$\bar{F} = \frac{\int_{t_1}^{t_2} F\,dt}{\Delta t}$$

## Center of mass

$$\boxed{CM = \frac{1}{M}\sum_{i=1}^{n} r_i m_i}$$

## Air resistance

Low velocity: $\vec{F}_r = -bv$

high velocity: $\vec{F}_r = -cv^2$

## Rocket motion

$$\boxed{F_{thrust} = \frac{dp}{dt} = V_{exhaust}\frac{dm}{dt}}$$

Collissions

Elastic collisions

$$\boxed{\begin{aligned} V_{1f} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) V_{2i} \\ V_{2f} &= \left(\frac{2m_1}{m_1 + m_2}\right) V_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) V_{2i} \end{aligned}}$$

notes; in the case $m_1 = m_2$,

$V_{1f} = V_{2i}$

in the case $m_2 >>> m_1$, $V_{2i} = 0$

$V_{1f} \approx -V_{1i}$

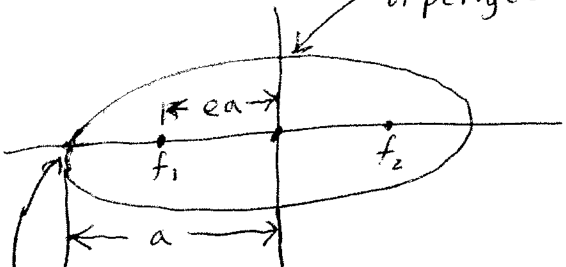
In elastic collissions, the relative velocities of the particles is equal and opposite to the relative velocities afterwards. ($\alpha = V_{1i} - V_{2i}$ is conserved)

# Gravitation

$$\boxed{\vec{F} = \frac{G m_1 m_2}{\vec{r}^3}\hat{r}}$$

## Kepler's 1st Law

perhelion or perigee




aphelion (if sun at focus)
apogee (if earth at focus)

## eccentricity

| | |
|---|---|
| $e = 0$ | circle |
| $0 < e < 1$ | ellipse |
| $e = 1$ | parabola |
| $e > 1$ | hyperbola |

## Kepler's 2nd Law

$$\frac{dA}{dt} = \lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} = \lim_{\Delta t \to 0} \frac{1}{2} r \cdot \frac{r\Delta\theta}{\Delta t} = \frac{1}{2} r^2 \omega$$

$$\boxed{L = mvr = mr^2\omega = \text{const.}}$$

## Kepler's 3rd Law

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

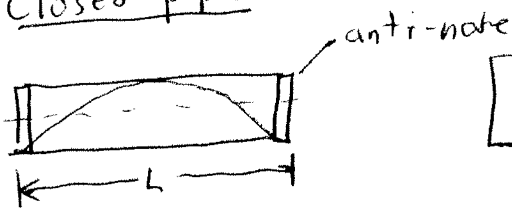
$$\boxed{T^2 \propto r^3}$$

# Sound Waves

$V_{sound, air} = 343\, m/s = 1\,mile / 5\,seconds$

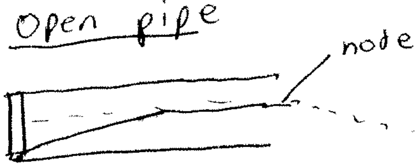
$$\boxed{V_{sound, air} \propto \sqrt{T}}$$

## Closed pipe




$$\boxed{\lambda = \frac{2L}{n}}$$ $n = 1, 2, 3, \ldots$

## Open pipe




$$\boxed{\lambda = \frac{4L}{n}}$$ $n = 1, 2, 3, \ldots$

## Doppler effect

$\nu_0$ = source frequency
$\nu'$ = observer frequency
$v$ = speed of sound
$v_s$ = speed of source
$v_o$ = speed of observer

master $$\boxed{\nu' = \nu_0 \frac{v \pm v_o}{v \mp v_s}}$$

observer at rest $$\boxed{\nu' = \nu_0 \frac{v}{v \mp v_s}}$$

# Angular Momentum

$$\boxed{L = m\vec{v}\times\vec{r} = \vec{p}\times\vec{r} = \vec{r}\times\vec{p} = mrv_{\perp} = I\omega}$$

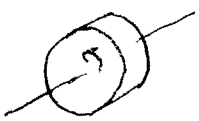
## Rotational Analogs

$$\boxed{K_{rot} = \frac{1}{2}I\omega^2}$$

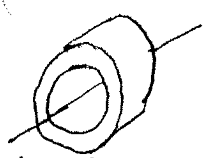
$$\boxed{\tau = I\alpha}$$

## Moments of Inertia

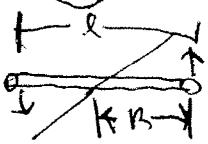
$$\boxed{I = mR^2}$$ hoop



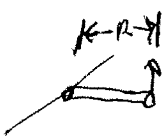
$$\boxed{I = \frac{mR^2}{2}}$$ disk / cylinder



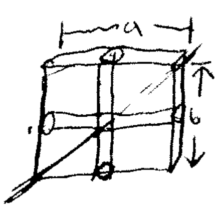
$$I = \frac{m(R_1^2+R_2^2)}{2}$$ hollow disk / cylinder



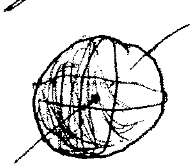

$$\boxed{I = \frac{mL^2}{12}}$$ *rod through center of mass* $\left(= \frac{mR^2}{3}\right)$




$$\boxed{I = \frac{mL^2}{3}}$$ *rod through one end*




$$I = \frac{m(a^2+b^2)}{12}$$ solid plate



$$\boxed{I = \frac{2}{5}mR^2}$$ sphere

## * Parallel Axis Theorem *

$$\boxed{I = I_{cm} + md^2}$$

# Classical Mechanics: Oscillations

## SHO

$$\ddot{x} + \frac{k}{m}x = 0$$

$$\omega^2 = k/m \quad \Rightarrow \quad \boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

## Simple pendulum

$$\tau = l\,mg\sin\theta = ml^2\ddot{\theta} = I\ddot{\theta}$$

$$\Rightarrow \ddot{\theta} + \frac{g}{l}\sin\theta = 0 \quad \text{or} \quad \ddot{\theta} + \frac{l\,mg}{I}\sin\theta = 0$$


$$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

## Physical pendulum

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgL}}}$$

## Forced/damped Oscillations

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f_0\cos\omega t$$

$$\Rightarrow \quad x(t) = A\cos(\omega t - \delta)$$

$$A = \sqrt{\frac{f_0^2}{(\omega_0^2-\omega^2)^2 + 4\beta^2\omega^2}}$$

$$\delta = \arctan\left(\frac{2\beta\omega}{\omega_0^2 - \omega^2}\right)$$

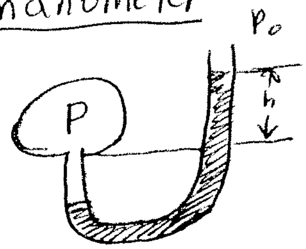
# Fluid statics

$$\boxed{P = \frac{dF}{dA}}$$

Pascal's principle : basis for hydraulics

Archimede's principle : bouyant force = weight of displaced water

## manometer




$$\boxed{P - P_o = \rho g h}$$

# Fluid Dynamics

## Continuity Equation

$$\boxed{\rho \vec{\nabla} \cdot \vec{v} + \frac{d\rho}{dt} = 0}$$

## Bernoolli's Equation

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

# Intermediate Mechanics

$L = T - U$

$H = T + V$

$$L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_k} = H - \sum_j p_j \dot{q}$$

## Lagrange's Euler Equation (w/ Lagrange multipliers)

$$\boxed{\frac{\partial L}{\partial q} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}} + \sum_k \lambda_k \frac{\partial f}{\partial q} = 0}$$

## Hamilton's Equations

$$\boxed{\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p} \\ \dot{p} &= -\frac{\partial H}{\partial q} \end{aligned}}$$

# E & M (18%)

~~curve-ball~~

## The Electric Field

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

$$\vec{D} = \varepsilon_0 \vec{E} = \vec{P}$$

## Electric susceptibility/Dielectric

$$E \propto \frac{1}{\varepsilon_0} \qquad \underline{\varepsilon = K\varepsilon_0}$$

$K$ = dielectric constant

## Electric Field of a Dipole

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}} \approx \frac{1}{4\pi\varepsilon_0}\frac{qd}{x^3} \quad \text{when } x >> d}$$

## Dipole moment

$$\boxed{\vec{p} = q\vec{d}}$$

## Electric dipole torque

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

## Electric Field of an Infinite Sheet

$$\boxed{\vec{E}_z = \frac{\sigma}{2\varepsilon_0}}$$

## Electric Potential Energy

$$\boxed{U(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

## Electric Potential

$$\boxed{V = \frac{U}{q_0}}$$

$$\boxed{\Delta V = \int_a^b \vec{E} \cdot d\vec{s} \iff \vec{E} = -\vec{\nabla} V}$$

## Capacitance

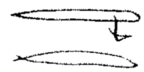
$$\boxed{C = \frac{Q}{V}}$$

## Parallel Plate Capacitor

$$\boxed{C = \frac{\varepsilon_0 A}{D}}$$

$$V = \int_a^b E \cdot ds = d \frac{q}{A \varepsilon_0}$$

$$=$$

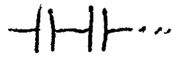


$$E = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A \varepsilon_0}$$

## Capacitors in Parallel

$$\boxed{C = C_1 + C_2 + \ldots}$$

## Capacitors in Series



$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots}$$

## Energy Stored in a Capacitor

$$\boxed{U = \frac{1}{2} C V^2} \boxed{= \frac{1}{2} \frac{Q^2}{C}}$$

## Capacitor w/ dielectric

$$\boxed{C \equiv K C_0}$$

## OHM's Law

$$\boxed{V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}}$$

## Resistivity

$$\boxed{\rho = \frac{\vec{E}}{\vec{J}} = \frac{\vec{E}}{i/A}} =$$

$$V = IR$$

$$\frac{V}{d} = \frac{I}{d} R$$

$$\boxed{\rho = R \frac{A}{\ell}}$$

$$\boxed{R = \rho \frac{L}{A}}$$

# Inductance

$$\boxed{\mathcal{E} = L\frac{di}{dt}} \xrightarrow{\text{Faraday's Law}} \boxed{L = \frac{\Phi}{i}}$$

## Energy of an inductor

$$\boxed{U = \frac{1}{2}Li^2}$$

$V_{out} = IR = \frac{V_{in}}{Z}R = \frac{VR}{R + i\omega L}$

as $\omega \to \infty$, $V_{out} \to$

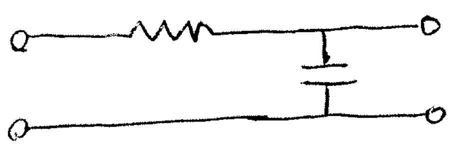
(~~high~~ low pass filter

~~$V_{out} = \frac{V_{in}}{Z}R = V_{in}R/R+$~~

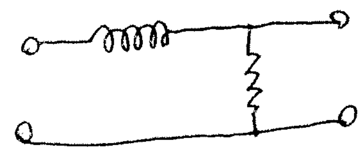
# Electric Filters

low pass filters

Vin

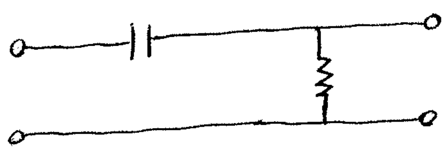


Vout

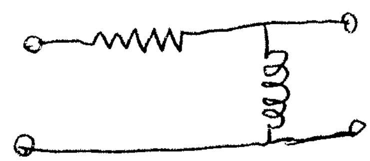


high pass filters

Vin



Vout



## Current Density

$$\boxed{j = \frac{i}{A} = V_d ne}$$

## Hall effect coefficient

$$R_H = -\frac{1}{nec} = \frac{E_y}{j_x B}$$

**Impedance:** $\boxed{V = IZ}$

defined as $V/i$; as a function of $\omega$. It is analogous to resistance with DC.

**Reactance:**

Is the complex part of impedance and results in a phase shift of the AC signal.

~~Resonance~~

## Impedance / AC Circuit Analysis

$\boxed{Z_{inductor} = i\omega L}$ $\boxed{Z_{capacitor} = \frac{-i}{\omega C}}$ $\boxed{Z_{resistor} = R}$

see next page →

~~...~~ Master equation: $\boxed{Z = R + i\omega L - i/\omega C}$ $\boxed{V = IZ}$

## Electric Potentials

$$\boxed{\vec{E} = -\nabla V - \frac{\partial A}{\partial t}}$$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

## Energy of E & B Fields

$$\boxed{U = \frac{1}{2}\int_V \left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) \partial V}$$

## Momentum of E & B Fields

$$\boxed{\vec{p} = \varepsilon_0 \int_V \vec{E} \times \vec{B}\, \partial V}$$

## Poynting Vector

$$\boxed{\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})}$$

## Biot - Savart Law

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\int I \frac{d\vec{\ell} \times d\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\int I \frac{d\vec{\ell} \times \vec{r}}{r^3}}$$

## Lamor Formula

$$\boxed{P \propto q^2 a^2}$$

# Maxwell's Equations

## Gauss's Law

$$\boxed{\oint_{\partial V} \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}}$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho}$$

## No Magnetic monopoles

$$\boxed{\int_{\partial V} \vec{B}\cdot d\vec{A} = 0}$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

## Faraday's Law

$$\boxed{\oint \vec{E}\cdot d\vec{\ell} = -\frac{\partial \Phi}{\partial t}}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}}$$

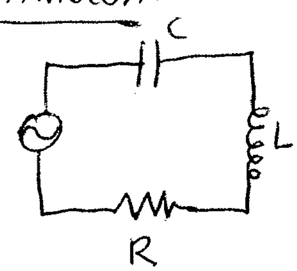
## Ampere's Law

$$\boxed{\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i_{enc} + \mu_0 \varepsilon \frac{\partial \Phi_E}{\partial t}}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}}$$

## AC Circuit Analysis continued...

### LRC Circuit




Resonance occurs when the reactance (complex impedance) is 0.
ie, $Z = R + i(\omega L - 1/\omega C) \Rightarrow \omega L = 1/\omega C \Rightarrow \omega^2 = \frac{1}{LC}$

$$\boxed{\omega = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}}$$

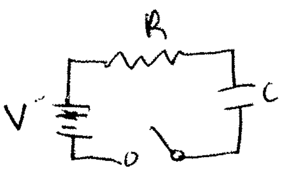
### Mechanical Analog

$m\ddot{x} + b\dot{x} + kx = 0$

$L\ddot{q} + R\dot{q} + \frac{q}{C} = 0$

## Circuit Analysis, continued

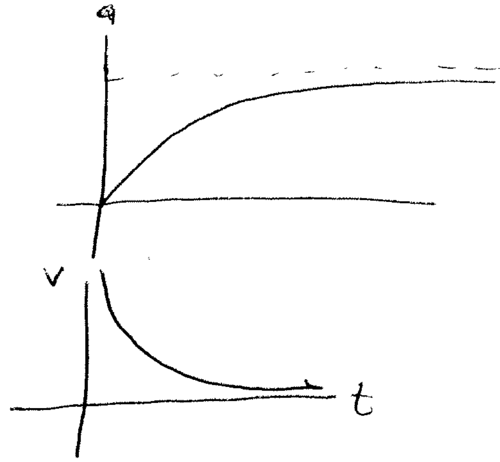
$V = IR$ $\boxed{\text{Power} = I^2 R}$

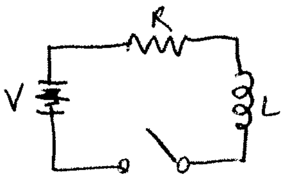



$\dot{q} R + \frac{q}{C} = V$

$q = CV\left[1 - e^{-t/RC}\right]$

$C = \frac{Q}{V}$




$\ddot{q} L + \dot{q} R = V$

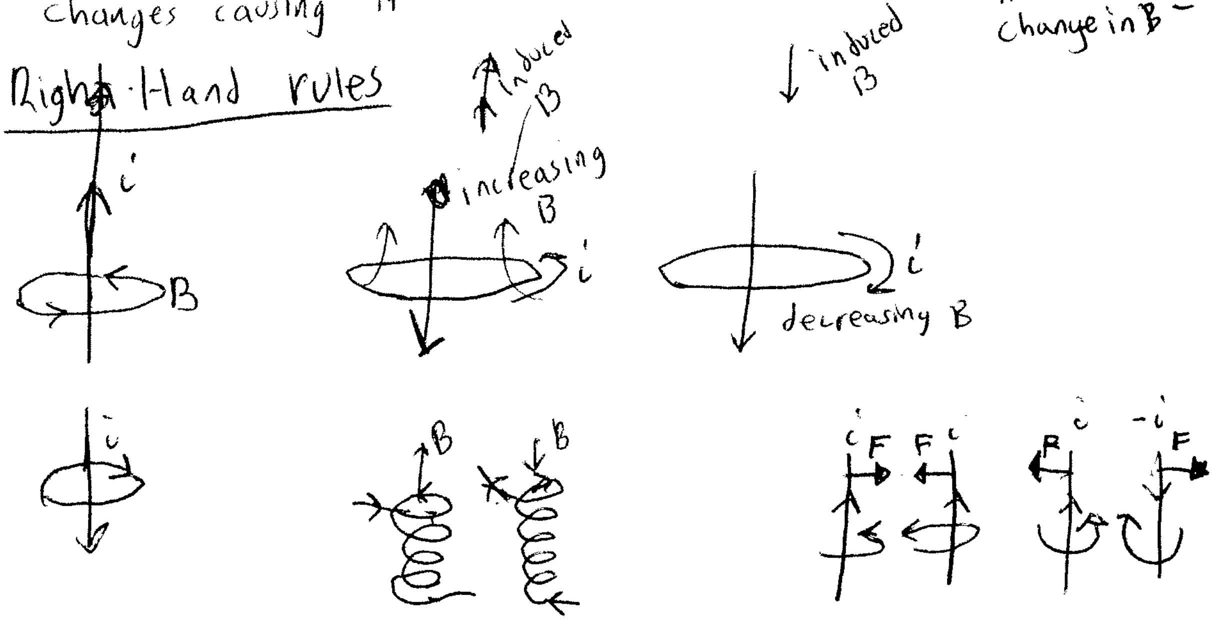
$\ddot{q} + \frac{\dot{q} R}{L} = V$

## Lenz's Law

"An induced current is always such to oppose the changes causing it"

- minimize change in B -

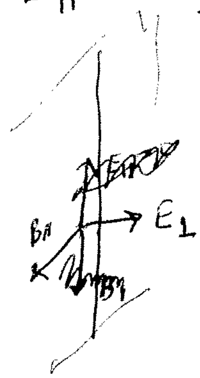
## Right-Hand rules

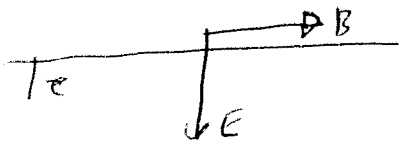


# More misc, E & M

~~Electric~~ Boundary conditions for reflection from an infinitely long, perfectly conducting sheet

$\vec{E}_{\parallel} = 0$ , $\vec{B}_{\perp} = 0$




For a perfect conductor, in equilibrium(3) there can never be an E-field parallel to the surface. The phase of an incoming wave shifts by π, so the E-field reverses direction, caneling out.


Griffiths pg 98

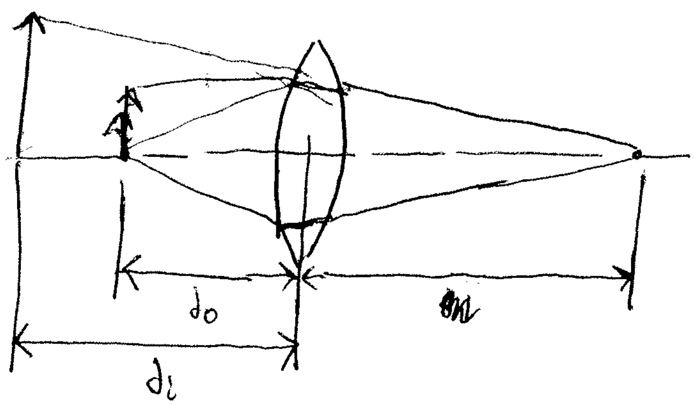
## Cyclotron frequency

$$qvB = \frac{mv^2}{r}$$

$$v = \frac{qrB}{m} = \frac{2\pi r}{T} f \Rightarrow \boxed{\omega = \frac{qB}{m}}$$

# Optics & Wave Phenomina (9%)

## Thin Lens Equation




$$\boxed{\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}}$$

$d_o$ = object distance
$d_i'$ = image distance

$$\boxed{M = -\frac{d_i}{d_o}}$$

$$\boxed{M = \frac{f_o}{f_e}}$$

## Rayleigh Criterion

$$\boxed{\theta = \frac{1.22\lambda}{D}}$$

## Malthus's law for Polarizers

$$\boxed{I(\theta) = I(0)\cos^2\theta}$$

## Light Intensity

$$\boxed{I \equiv \varepsilon_0 c \langle E^2 \rangle_t}$$

## Snell's Law

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

## Total internal reflection

$$\boxed{\theta = \arcsin\left(\frac{n_2}{n_1}\right)}$$

## Brewster's angle

$$\boxed{\theta_B = \arctan\left(\frac{\theta_2}{\theta_1}\right)}$$

## Optical Path Length

$$\boxed{OPL = \int_c n\, d\ell} = \sum_i n_i d_i$$

## Maxwell's Relation

$$\boxed{n \approx \sqrt{K}}$$

## Double slit interference /

$s = \theta r \quad \theta = \frac{s}{r} \approx \tan\theta$

$$\boxed{d\sin\theta = m\lambda} \quad m = 0, 1, 2, \ldots \quad \text{(maxima)}$$

$\approx \sin\theta \approx \theta \approx \frac{y}{D}$

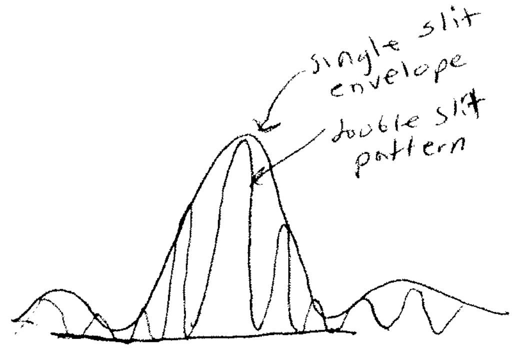
$y \approx \frac{m\lambda D}{d}$ for maxima

Phase change after reflectance $\approx \pi$ (from lower to higher $n$)

## Bragg Diffraction

Waves are in phase when

$$\boxed{2d\sin\theta = n\lambda} \quad n = 0, 1, 2, \ldots$$




## Other diffraction problems

Diffraction Grating as well

1-slit diffraction minima

width of slit

$$\boxed{w\sin\theta = m\lambda}$$

double-slit diffraction

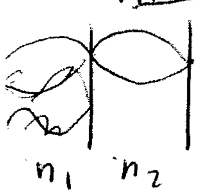
minima $\quad w\sin\theta = m\lambda$

"missing fringes" - occurs when diffraction minima cancel interference maxima in the double slit experiment

Circular diffraction, first minima

$$\sin\theta \approx \frac{1.22\lambda}{D}$$ (same as Rayleigh criterion)

## Thin Films




general principles:
phase change occures when $n_2 > n_1$, Does not occur when $n_2 < n_1$

constructive interference $\quad 2t = \frac{\lambda}{2}$

destructive interference $\quad 2t = \lambda$

# Thermodynamics : 10%

## Heat Capacity

$$\boxed{C = \frac{dQ}{dT}}$$

## Specific Heat Capacity

$$\boxed{c = \frac{1}{m}\frac{dQ}{dT}}$$

## Mayer's Equation

$$\boxed{C_p - C_v = R = Nk}$$

## Ideal Gas Law

$$\boxed{PV = nRT} \quad \boxed{PV = NkT}$$

## Work

$$\boxed{W = -\int P\,dV}$$ general equation

constant volume ; $W = 0$

constant pressure : $W = -P(V_f - V_i)$

adiabadic : $PV^\gamma = \text{const.} \longrightarrow W = \frac{1}{\gamma - 1}(P_f V_f - P_i V_i)$ $\quad \gamma = \frac{c_p}{c_v}$

Constant temperature $W = -nRT \ln\left(\frac{V_f}{V_i}\right)$

## Thermal Expansion

$$\boxed{\Delta L = \alpha L \Delta T}$$

## Speed of sound

$$\boxed{V_s \propto T^{1/2}}$$

## Entropy

$$\boxed{ds = \frac{dQ}{T}}$$

## Boltzmann's Law

$$\boxed{S = k \log w}$$

$w =$ # of possibilities

## Law of Dulong & Petit

$$\boxed{C_v = 3R = 3Nk}$$ at high $T$

## Debye $T^3$ Law

$$\boxed{C_v \propto T^3}$$ at lower $T$

## *Equipartitión Theorem*

$$\boxed{E = \frac{1}{2} n k T}$$

$n =$ DOF

for a solid, $n = 6$ → 3KE, → 3PE

## Specific heat of a metal

$$\boxed{c = aT + BT^3}$$

$c = e^{aT}$ (superconducting)

## Stirling's Theorem / approximation

$$\ln N! \approx N \ln N - N \qquad N \gg 0$$

# Thermodynamics pg 2

## Internal energy of a gas (equi partition theorem)

$$\boxed{U = \left(\frac{1}{2} nkT\right) N}$$

n = degrees of freedom
N = number of atoms/molecules

## Van der Waals equation

$$\left(p + a\frac{n^2}{V^2}\right)\left(V - nb\right) = nRT$$

## Maxwellian speed distribution

$$n(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

## Maxwell-Boltzmann energy distribution

$$n(E) = \frac{2N}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} E^{1/2} e^{-E/kT}$$

## Bose-Einstein distribution

$$\boxed{f_{BE}(E) = \frac{1}{e^{(E-E_0)/kT} - 1}}$$

indistinguishable
no Pauli Exclusion

## Fermi-Dirac distribution

$$\boxed{f_{FD}(E) = \frac{1}{e^{(E-E_0)/kT} + 1}}$$

indistinguishable
Pauli Exclusion principle

## Boltzmann Distribution / Maxwell-Boltzmann

$$\boxed{f_j = \frac{Ne^{-\varepsilon/kT}}{\sum_j g_j e^{-\varepsilon_j/kT}}}$$

distinguishable

## Blackbody Planck Formula

$$\boxed{\frac{\text{Power}}{\text{Area}\cdot\lambda} = \frac{\text{Flux}}{\partial\lambda} = \frac{\partial R}{\partial\lambda} = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}}$$

## Stephan-Boltzman Law

$$\boxed{R = \text{Power Output} = \sigma T^4}$$

## *Wein's Law for Blackbody*

$$\boxed{\lambda_{max} \propto \frac{1}{T}}$$

## First Law of Thermodynamics

A system goes from state $i$ to $f$ through various paths, the quanity $Q+W$ is always the same.

$$\boxed{dE = dQ + dW}$$

## Second Law of Thermodynamics

There are no perfect heat engines.

↳ It is not possible for a cyclical process to convert heat entirely to work.

↳ It is impossible to build a heat engine more efficient then a carnot engine. $\boxed{e_{carnot} = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H}}$ *

↳ A perfectly reversible engine has carnot efficiency.

It is impossible to reach absolute zero. (Sometimes called "3rd Law")

## Zeroth Law of thermodynamics

If A & B are in T, E, ~~then~~ with C,
then A and B are in T, E, with each other.

## Fourier's Law of Heat Conduction

$$\boxed{\vec{q} = -k \nabla T}$$

$k$ = thermal conductivity

$q$ = heat flux $= \frac{dQ}{dt}\frac{1}{A}$

$$\boxed{\frac{dQ}{dt} = -kA\frac{dT}{dX}}$$

## ~~Va~~ 1st Law of thermodynamics / Energy

$$dE = dQ + dW$$

$$\boxed{dU = TdS - PdV + \mu\, dn}$$

($\mu$ = chemical potential)

## The Partition Function

$$\boxed{Z \equiv \sum_j g_j e^{-\varepsilon_j/kT}}$$

$$S = \frac{U}{T} + Nk(\ln Z - \ln N + 1)$$

$$F = -NkT(\ln Z - \ln N + 1)$$

$$\boxed{U = NkT^2\left(\frac{\partial \ln Z}{\partial T}\right)}$$

$$H = U + PV \qquad dH = TdS - PdV$$

$$F = U - TS \qquad dH = TdS + VdP$$

## RMS Speed: quick derivation

$$\frac{3}{2}kT = \frac{1}{2}mv^2$$

$$\hookrightarrow \boxed{V_{rms} \approx \sqrt{\frac{3kT}{m}}}$$

## ~~Diffusion~~ Graham's Law of Effusion

$$\frac{rate_1}{rate_2} = \sqrt{\frac{M_2}{M_1}} \leftarrow \text{molar mass}$$

can be derived by simply noting that $V_{RMS} \propto \sqrt{\frac{1}{m}}$

# Quantum Physics (12%)

## The Schrödinger Equation

$$\boxed{\frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}}$$

## Operators

position $\hat{x} = x$

momentum $\hat{p} = -i\hbar\frac{\partial}{\partial x}$

Energy $\hat{E} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$

## Debroglie Formulae

$E = \frac{p^2}{2m}$

$p = \sqrt{2mE}$

$$\boxed{p = \hbar k = \frac{h}{\lambda}}$$

## Planck Formulae

only applies to photons!

$$\boxed{E = \hbar\omega = hf = \frac{hc}{\lambda}}$$

## Heisenberg's Uncertainty Relations

$$\boxed{\sigma_x\sigma_p \geq \frac{\hbar}{2}} \qquad \boxed{\sigma_t\sigma_E \geq \frac{\hbar}{2}}$$

## Ehrenfest's Theorem

$$\boxed{\frac{d\langle p\rangle}{dt} = -\frac{\partial V}{\partial x}}$$

## Compton Scattering Formula

$$\boxed{\Delta\lambda = \frac{hc}{mc^2}(1-\cos\theta)}$$

$$\lambda_{compton} = \frac{hc}{mc^2} = \frac{h}{mc}$$

## TISE

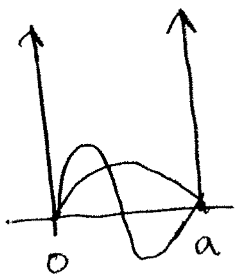
$$\boxed{\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi}$$

## Time Dependence

$$\boxed{f(t) = e^{-iEt/\hbar}}$$

# Quantum Physics Continued

## Infinite Square Well




$$\boxed{E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2ma^2}}$$

$$\boxed{\Psi = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right) e^{-iEt/\hbar}}$$

## The Harmonic Oscillator

$$\frac{-\hbar^2}{2m} \nabla^2 \Psi + V\Psi = E\Psi$$

$$\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi + \frac{1}{2} m\omega^2 x^2 \Psi = E\Psi \qquad \omega^2 = \frac{k}{m}$$

$$\frac{1}{2m}\left[\left(\frac{\hbar}{i}\frac{d}{dx}\right)^2 + (m\omega x)^2\right]\Psi = E\Psi$$

$$a_\pm \equiv \frac{1}{\sqrt{2m}}\left(\frac{\hbar}{i}\frac{d}{dx} \pm im\omega x\right)$$

$$\left(a_- a_+ - \frac{1}{2}\hbar\omega\right)\Psi = E\Psi$$

$$\vdots$$

$$\Psi_0 = A_0 e^{-\frac{m\omega}{2\hbar} x^2}$$

$$\boxed{E_n = \left(n + \frac{1}{2}\right)\hbar\omega}$$

## The Free Particle:

$$\Psi(x,t) = A e^{ik\left(x - \frac{\hbar k}{2m} t\right)} + B e^{-ik\left(x + \frac{\hbar k}{2m} t\right)}$$

Fourier transform:

$$\boxed{\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x,0) e^{-ikx} dx}$$

Group Velocity

$$\boxed{V_{group} = \frac{\partial \omega}{\partial k}}$$

Phase Velocity

$$\boxed{V_{phase} = \frac{\omega}{K}}$$

Delta Function

$$\boxed{\int_{-\infty}^{\infty} f(x)\,\delta(x-a)\,dx = f(a)}$$

Basic Definitions

Hermition operator ; $T^{+} = T$

Unitary operator : $U^{+} = U^{-1}$

Probability Current

$$\frac{d P_{ab}}{dt} = J(a,t) - J(b,t)$$

$$\boxed{J(x,t) \equiv \frac{i\hbar}{2m}\left(\Psi \frac{\partial \Psi^*}{\partial t} - \Psi^* \frac{\partial \Psi}{\partial t}\right)}$$

Commutator Relations

$$\boxed{[A, B] = [-B, A]}$$

B, AC = BAC − ACB

A·[BC] + [B, A] C

ABC − ACB + BAC − ABC

$$\boxed{\begin{aligned} [A, B] &= AB - BA \\ [AB, C] &= A[B, C] + [A, C]B \end{aligned}}$$

# Spin & Angular Momentum

$$\boxed{\begin{aligned} L^2\psi &= \hbar^2 \ell(\ell+1)\psi \\ L_z\psi &= m\hbar\,\psi \end{aligned}}$$

# Pauli matrixes

$$\boxed{\begin{aligned} \sigma_1 = \sigma_x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma_2 = \sigma_y &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma_3 = \sigma_z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned}}$$

# Hydrogen atom

$$\boxed{E = -13.6\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

to derive, assume that $m_e v r = n\hbar$

Bohr radius $= a_0 = \dfrac{4\pi\varepsilon_0\hbar^2}{me^2}$

# Angular Momentum quantum numbers

$n$ : principal ~~q~~quantum number –

$\ell$ : Azimuthal quantum number – gives orbital angular momentum

$m_\ell$ : magnetic quantum number, ranges from $-\ell$ to $\ell$

$s$ : spin quantum number

for electrons $= \{1/2, -1/2\}$ $\quad S_z = m_s\hbar$

$|\vec{S}| = \sqrt{s(s+1)}\,\hbar$ ~~for~~ $= m_s\hbar$

$= \frac{\sqrt{3}}{2}\hbar$ for electrons,

$m_s$ goes from $-s$ to $s$ in $\mathbb{Z}$

$2(2\ell+1)$

| $\ell$ | | $\#e^- = \cancel{\ldots} = 4\ell+2$ |
|---|---|---|
| 0 | s | 2 |
| 1 | p | 6 |
| 2 | d | 10 |
| 3 | f | 14 |

# Misc. Modern Physics Equations

## Positronium

$$\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e^2}{2m_e} = \frac{m_e}{2} \Rightarrow$$ E levels are 1/2 of hydrogen's.

## The Planck Length

$$\boxed{L_p = \sqrt{\frac{G\hbar}{c^3}}}$$

## Schwartzchild Radius

$$\boxed{R_s = \frac{2MG}{c^2}}$$

## Constants

$$\boxed{kT \approx 1/40 \text{ eV at } 300k}$$

$$\boxed{hc = 1240 \text{eV}\cdot\text{nm}}$$

$G = 6.67 \times 10^{-11}$

$k = 8.62 \times 10^{-5}$ eV/K

## Photoelectric Effect

$$\boxed{E = eV_{stop} = hf - \phi}$$

## Bohr Atom

$$\boxed{\Delta E = -Z^2 13.6 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

## Spectroscopic Notation / "Term Symbol"

$${}^{2s+1}L_J$$

s = spin #

$L = \ell =$ 0 = s, 1 = p, 2 = d, ...

$L = \ell =$ s, p, d, f

$\vec{J}$ = total angular momentum

$$\boxed{\vec{J} = \vec{S} + \vec{L}}$$

$L = \ell_1 + \ell_2 + ..$

$S = s_1 + s_2 + ..$

for example

${}^3S$

refers to

$2s+1 = 3$

$s = 1$

$J = 0 + 1 = 1$

# Misc.

## Intrinsic Magnetic Moment

$$\vec{\mu}_s = \gamma \vec{s}$$

$$\gamma = \text{gyromagnetic ratio} = \frac{eg}{2m}$$

## A useful rule of thumb for Energy levels

Electronic transitions $\approx 1\,eV$

Vibrational transitions $\approx .1\,eV$

Rotational transitions $\approx .001\,eV$

## Special Relativity (6%)

$$\boxed{\gamma = \frac{1}{(1-\beta^2)^{1/2}}} \qquad \boxed{\beta = \frac{v}{c}}$$

$\gamma(\beta = 1/2) = 1.15$

$\gamma(\beta = .9) = 2.3$

## Energy-momentum

$$\boxed{E^2 = p^2c^2 + m^2c^4}$$ * master equation *

$$\boxed{E = \gamma mc^2} \qquad \frac{mc^2}{(1-1} \qquad \boxed{KE = \gamma mc^2 - mc^2}$$

$$\boxed{p = \gamma mv}$$

for a photon, $m=0$ and

$$\boxed{E = pc}$$

## Lorentz transformation

$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{xv}{c^2}\right) \end{aligned}}$$

## Transformation of velocities *

$u = \frac{dx}{dt}$ = velocity in frame S

$u' = \frac{dx'}{dt'}$ = velocity in frame S'

S' is moving with velocity V

$$\boxed{u' = \frac{u - V}{1 - \frac{uV}{c^2}}}$$

## * The invariant / spacetime interval * proper distance *

$$\boxed{ds^2 = dr^2 - c^2 dt^2}$$

4 vectors: $X_a = \begin{pmatrix} ct \\ \vec{x} \end{pmatrix} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$

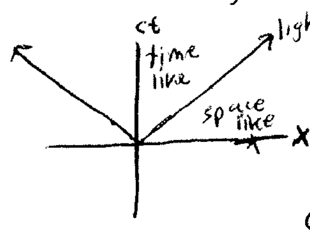
dot product $V_1 \cdot V_2 = ct_1 ct_2 - \vec{x}_1 \cdot \vec{x}_2$

Energy - momentum 4 vector $\vec{p} = \begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix}$

The length of 4-vectors is invariant.

## * Time dilation *

$$\boxed{t = \gamma t_0}$$




$c^2 \Delta t^2 > \Delta r^2$ time like interval
- same location possible
- but not same time

$c^2 \Delta t^2 = \Delta r^2$ light like interval

$c^2 \Delta t^2 < \Delta r^2$ space-like interval
- same time possible
- but not same location

## * Length contraction *

$$\boxed{l = \frac{l_0}{\gamma}}$$

## The Doppler Shift

$E' = \gamma E_0 + \beta \gamma E =$

Blue Shift: $\boxed{\frac{\lambda'}{\lambda} = \sqrt{\frac{1-\beta}{1+\beta}}}$

Red Shift: $\boxed{\frac{\lambda'}{\lambda} = \sqrt{\frac{1+\beta}{1-\beta}}}$

## Standard deviation

$$\sigma^2 = \overline{(x-\bar{x})^2} = \langle (x - \langle x \rangle)^2 \rangle = \boxed{\langle x^2 \rangle - \langle x \rangle^2}$$

## Poisson distribution (random distribution)

$$\boxed{\sigma \approx \sqrt{\bar{x}}}$$

## Propagation of errors

$$* \quad \boxed{\delta x = [\delta x_1^2 + \delta x_2^2 + \ldots]^{1/2}} \quad *$$

Random errors : can be seen during multiple measurements and ~~and~~ thus accounted for

Systematic errors : errors that are intrinsic to the instrumentation and cannot be revealed by repeated measurement.

# Particle Physics (3%)

## Hadrons: composite particles made of quarks

↳ **Mesons**: 2 quarks

$\pi^+, \pi^0, K^+, K^0, \psi$

↳ **Baryons**: 3 quarks

$p = uud$
$n = udd$
$\Lambda^0, \Sigma^+$

**Baryon #** = +1 for baryons,
−1 for anti-baryons
0 for non-baryons

## Fermions: 1/2 integer spin

↳ **Leptons: spin 1/2**

$e^-, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$

There are 3 lepton #s

$L_e, L_\mu, L_\tau = \{+1, -1, 0\}$

↳ **Quarks: spin 1/2**

| | | |
|---|---|---|
| +2/3 U | +2/3 C | +2/3 T |
| −1/3 D | −1/3 S | −1/3 B |

## Bosons: integer spin particles

$\gamma$, g, graviton, $W^+, W^-, Z^0$

Conservation Laws:

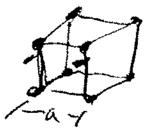
Energy & momentum
Parity - not in weak
charge
Time symetry
Strangeness - not in weak
Baryon #
Lepton #

# Solid State Physics (~3/1)

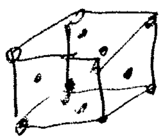
## The Primitive Cell

$$\boxed{V_{\text{primitive cell}} = \frac{V_{\text{Bravais lattice}}}{\#\text{ of lattice points}}}$$


simple cubic $\rightarrow$ $V_p = a^3$
1 lattice point inside




B.C.C. $\rightarrow$ $V_p = a^3/2$
2 lattice points



F.C.C. $\rightarrow$ $V_p = a^3/4$
4 lattice points

## Resistivity of a semiconductor

$$\boxed{\rho \approx 1/T}$$

## Effective Mass

$$\boxed{m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}}$$

"really simple" derivation:

$p = \hbar k$

$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$

$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$ $\quad \frac{1}{m} = \frac{d^2E}{dk^2}\frac{1}{\hbar^2}$

$m = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$

# Math $(\sim 3\%)$

mapping

gradient $\nabla f = \left( \frac{\partial f_1}{\partial x_1}, \dots, \frac{\partial f_n}{\partial x_n} \right)$ $\quad \mathbb{R}^n \to \mathbb{V}^n$

divergence $\vec{\nabla} \cdot \vec{F} = \frac{\partial F_1}{\partial x_1} + \dots + \frac{\partial F_n}{\partial x_n}$ $\quad \mathbb{V}^n \to \mathbb{R}$

curl (in $\mathbb{V}^3$) $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}$ $\quad \mathbb{V}^n \to \mathbb{V}^n$

## Divergence Theorem

$$\int_{\partial V} \vec{F} \cdot \partial S = \int_V \vec{\nabla} \cdot \vec{F} \, \partial V$$

## Stoke's Theorem (basic form for physics)

$$\int_{\partial A} \vec{F} \cdot \partial \vec{r} = \int_{A} \vec{\nabla} \times F \cdot \partial \vec{A}$$

## Important Identities

$$\nabla \cdot (\nabla \times \vec{H}) = 0$$

$$\nabla \times (\nabla f) = 0$$

## BAC - CAB rule

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$$

# Basic Equations from Astronomy / Astrophysics (~1/1)

$$\boxed{m_1 - m_2 = -2.5 \log_{10}(F_1/F_2)}$$ Apparent magnitude

$$M = m + 5 - 5 \log_{10}(d/pc) - A$$ Absolute magnitude

(interstellar extinction)

$$\text{fov} \propto \frac{1}{f}$$

$$\boxed{\text{Magnification} = f_o / f_{eyepiece}}$$

$$\boxed{\alpha_c = \frac{1.22\lambda}{D}}$$ angular resolution / Rayleigh criterion

$$\text{Image Scale} = \frac{1}{f_o \tan 1''} = \frac{206{,}265}{f} \Rightarrow \frac{\text{arc seconds}}{\text{mm}}$$

# Basic tips

→ If a problem looks hard / tedious, try process of elimination first. Use limiting cases to eliminate answers.

→ Very few problems are simply "plug into this equation", although a few are. Most problems combine at least 2 concepts or have a twist ~~or~~ to them.

→ Dimensional analysis is ocassionally useful.

## Statistics from past Exams

| Exam | # needed to get 700 | % got 700 | # 800 | % got 800 | # to get 900 | % 900 | # 990 | % 990 |
|---|---|---|---|---|---|---|---|---|
| GRE 8677 1981-1984 | 45 | 33% | 59 | 15% | 72 | 5% | 84 | 2% |
| GRE 9677 1993-1996 | 32 | 39% | 44 | 21% | 56 | 9% | 67 | 3% |
| GRE 9277 1988-1991 | 40 | 42% | 53 | 23% | 65 | 10% | 76 | 3% |
| GRE 0177 2000-2003 | 44 | 41% | 58 | 22% | 73 | 10% | 85 | 2% |
| Average | 41 | 39% | 53 | 20% | 67 | 8.5% | 78 | 2.5% |