Using Third-Order Moments of Fluctuations in V and B to Determine Turbulent Heating Rates in the Solar Wind

M.A. Forman[1], C.W. Smith[2], B.J. Vasquez[2], B.T. MacBride[3], J.E. Stawarz[2], J.J. Podesta[4], D.C.Elton[5], U.Y.Malecot[6], Y. Gagne[7]

1. Stony Brook University, NY11794-3800 USA; 2. Space Science, Institute for Earth, Oceans and Atmospheres, University of New Hampshire, Durham New Hampshire, NH 03824 USA; 3.Physics Department, University of California, Berkeley, CA94720-7300, USA; 4. Space Science and Applications (ISR-1),Los Alamos National Laboratory, Los Alamos, NM 87545, USA; 5. Physics Department, Rensselaer Polytechnic Institute, NY 12180,USA; 6. University Joseph Fourier, Grenoble 1, BP53,38041 Grenoble Cedex9, France; 7.Laboratoire des Ecoulements Geophysiques et Industriels, CNRS/UJF/INPG UMR5519, BP5338041 Grenoble, France

Abstract. The inertial range scaling of certain mixed third-order moments of velocity and magnetic field fluctuations in a turbulent MHD plasma such as the solar wind is related to the energy dissipation rate of the turbulence. We have used this relation to measure energy dissipation rates in the solar wind and other statistical methods to estimate the accuracy of these measurements. This paper reviews results we and others have recently published, and some new results.

Keywords: MHD, Turbulence, Theory, Solar Wind,
PACS: 96.50.Tf (MHD waves; plasma waves, turbulence); 96.50.Bh (Interplanetary magnetic fields)

INTRODUCTION

Fluctuations in solar wind velocity and magnetic field on scales of ten seconds to hours are thought to be part of a turbulent cascade of energy and cross-helicity from larger to smaller scales. In his theory of homogeneous and incompressible isotropic hydrodynamic turbulence, Kolmogorov derived from the Navier-Stokes equations the exact "4/5 law" that the (signed) third-order moment of the velocity increment in direction L, on scale L, $<(\Delta V_{\|}(L))^3> = -0.8\,\varepsilon L$, where ε is the energy injection rate. [Note: this is NOT the third moment used in intermittency studies of the slope of the structure function. Such work uses the absolute value of fluctuations, not the signed fluctuations, and is larger in magnitude.] The power spectrum of the fluctuations is predicted to be proportional to $k^{-5/3}$, but that alone does not indicate the presence of a turbulent energy cascade.

The objective of our work reviewed here is to verify that the relevant MHD third moments in the solar wind are not zero and do scale linearly with lag, as they would in an inertial cascade (but not in a field of randomly phased waves with the same power spectrum), and to then apply the complete anisotropic MHD version of the Kolmogorov 4/5 law to determine the energy cascade rate in the solar wind.

THIRD MOMENT IN MHD TURBULENCE THEORY

Politano and Pouquet (1998) showed that for homogeneous, incompressible and isotropic MHD turbulence **in the inertial range**, the MHD equations imply that the pair of vector third moments of fluctuations, $\boldsymbol{D3}^{\pm}(\boldsymbol{L}) \equiv \langle \Delta\boldsymbol{Z}^{\mp}(\boldsymbol{L}) |\Delta\boldsymbol{Z}^{\pm}(\boldsymbol{L})|^{2}\rangle$ at lag vector **L** obey the pair of vector differential equations

$$\nabla_L \cdot \boldsymbol{D3}^{\pm}(\boldsymbol{L}) = -4\varepsilon^{\pm}, \qquad (1)$$

where $\varepsilon^{\pm}$ are the pseudoenergy dissipation rates of fluctuations in the two Elsasser variables $\boldsymbol{Z}^{\pm} \equiv \boldsymbol{V} \pm \boldsymbol{B}/\sqrt{4\pi\rho}$. Taking differences in the solar wind parameters measured at a single spacecraft, $\Delta\boldsymbol{Z}^{\pm}(\boldsymbol{L}) = -[\boldsymbol{Z}^{\pm}(t+V\tau) - \boldsymbol{Z}^{\pm}(t)] = -\Delta\boldsymbol{Z}^{\pm}(\tau)$.

The two equations (1) mix the fluctuations in plasma velocity and magnetic field but can be re-combined to yield separate equations for the energy dissipation rate and the dissipation rate of cross helicity, using

$$\boldsymbol{D3}(\boldsymbol{L}) \equiv \tfrac{1}{2}\left(\boldsymbol{D}3^{+}(\boldsymbol{L}) + \boldsymbol{D3}^{-}(\boldsymbol{L})\right) \qquad (2)$$

and $\boldsymbol{D3^{CH}}(\boldsymbol{L}) \equiv \tfrac{1}{2}\left(\boldsymbol{D3}^{+}(\boldsymbol{L}) - \boldsymbol{D3}^{-}(\boldsymbol{L})\right)$.

Note that $\boldsymbol{D3}$, which may be called the energy flux in lag space, is even in magnetic fluctuations, but has a

CP1216, *Twelfth International Solar Wind Conference,*
edited by M. Maksimovic, K. Issautier, N. Meyer-Vernet, M. Moncuquet, and F. Pantellini

cross-helicity term. We have calculated these vectors and estimated the energy dissipation rates in different classes of solar wind turbulence using Elsasser variables from all of the data from the ACE spacecraft at L_1 in the ecliptic, and lately with some Ulysses data at high latitudes in 1996 [see REFERENCES].

Isotropic and Hybrid Data Analysis Methods

If the turbulence is isotropic, only the radial components enter equation (1), and the $\epsilon^{\pm}$ in the solar wind can be determined from the isotropic scaling with τ: ${}^{ISO}D3^{\pm}(\tau) \equiv \langle \Delta Z^{\mp}{}_{R}(\boldsymbol{L})|\Delta \boldsymbol{Z}^{\pm}(\boldsymbol{L})|^{2}\rangle = \frac{4}{3}\epsilon^{\pm}V\tau$. Because solar wind turbulence is not isotropic, it is important to correctly handle the anisotropy in D3 when estimating $\epsilon^{\pm}$. Also, it is interesting to examine the anisotropy in the vector field **D3(L)** as a clue to how the power spectrum anisotropy arises. If we may at least assume that SW turbulence is axi-symmetric around the local mean magnetic field, with separate dependence on the lag components parallel and perpendicular to the local magnetic field, then we may write $\boldsymbol{D}3(\boldsymbol{L}) = {}^{2D}D(\boldsymbol{L})\hat{e}_{\perp} + {}^{1D}D(\boldsymbol{L})\hat{e}_{\parallel}$, where the unit vector $\hat{e}_{\perp}$/ $\hat{e}_{\parallel}$ is along the solar wind component perpendicular/parallel to the local mean magnetic field. We have looked at the scaling of the perpendicular component with perpendicular lag, and of the parallel component with parallel lag in MacBride, et al (2008) and Stawarz, et al. (2009). To the extent that each D is linear in the lag in that direction, the hybrid expression is

$$\varepsilon = \left(\frac{{}^{2D}D(\tau)}{2V_{\perp}\tau} + \frac{{}^{1D}D(\tau)}{4V_{\parallel}\tau}\right).$$

Methods and results of this method are discussed at length in MacBride et al. (2005, 2008), Podesta, et al. (2007) and Stawarz, et al. (2009), but are outside the scope of this paper.

Uncertainty in estimates of the third moment: calculating "error bars"

We were at first surprised that several months of 64-second ACE data was usually needed for the calculated third moments to converge (i.e., stabilize with the addition of more data), forcing us to derive a method to test convergence to a reliable third moment and to find how to estimate its uncertainty. In MacBride et al. (2005, 2008), we estimated the best value by calculating third moments for 100 or more 2-day subsets and taking their average. In Stawarz et al. (2009), we use 12-hour subsets and again take their mean as the best value of the third moment, and estimate the uncertainty in the third moment as the standard error of the mean, σ(D3)/√N.

In Podesta, et al. (2009) we investigate the mathematics of convergence of third moments, by sub-setting data and estimate the uncertainties or error bars of third-order moments computed from experimental data. The ratio $\frac{\sigma(D3)}{|\langle D3\rangle|}$ for various turbulent HD and ACE velocity data sets decreases as $\frac{constant}{\sqrt{N}}$ where N is the number of data points *per subset*, and the constant is a few hundred, so that about 10^6 data points are needed for 30% accuracy (Podesta, et al. 2009; Stawarz, et al. 2009). This large number is characteristic of turbulence in the solar wind in the ecliptic at 1 AU and for determining *signed* third moments of certain wind-tunnel turbulence.

Without dividing into sub-sets, the uncertainty in $\langle \boldsymbol{D3}\rangle$ ***is*** $\sqrt{\frac{\langle \boldsymbol{D6}\rangle}{\boldsymbol{N_C}}}$ where D6 is the sixth moment and N_C is the number of correlation lengths of $(\Delta V)^3$ in the whole data set (Podesta,et al. 2009; Stawarz, et al. 2009). Because of intermittency and other effects, the sixth moment is large and many data points are needed for a reliable estimate of D3. For ACE 64-second data over 200,000 points are required.

RESULTS

1. Solar Wind in the Ecliptic at 1 AU

As shown in figure 1, the scaling of the third moment for energy is close to linear in lag in the inertial range as theory predicts (we studied 64 seconds to 2 hours), confirming that the fluctuations are not random, but part of a direct cascade of turbulent energy from larger to smaller scales (MacBride, et al. 2008).

As also shown in figure 1 and in figure 2, the third moment for energy is **anisotropic**, with the parallel cascade dominating in slow solar wind and the perpendicular cascade dominating in fast solar wind (MacBride, et al., 2008).

The average turbulent energy cascade rate shown by year in figure 3 did not vary greatly from 1998 to 2004. (MacBride, et al. 2008) As shown in figure 4, the energy dissipation from the turbulent cascade, presumably heating the solar wind locally, increases linearly with $V_{sw}T_{sw}$, where T_{sw} is the solar wind temperature, and agrees closely with the values inferred in studies of the radial variation of solar wind temperature (Stawarz, et al., 2009).

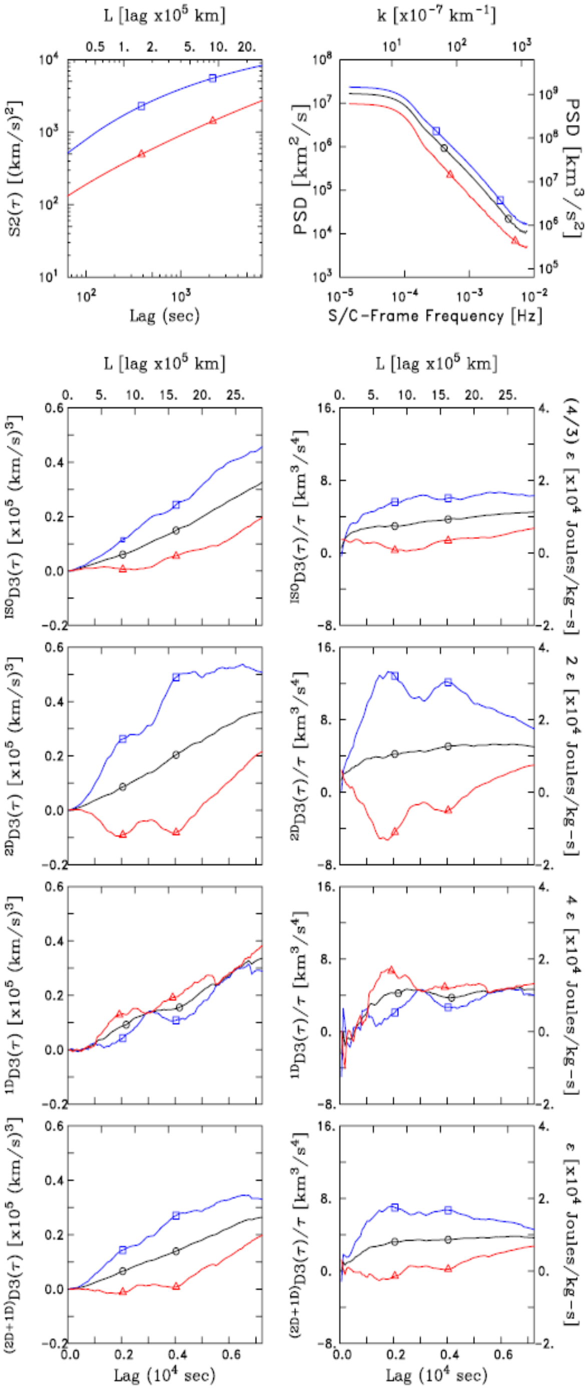

FIGURE 1. Lower four rows: Mixed third moments of velocity and magnetic fluctuations in solar wind in 1998-2004 measured at ACE in the solar wind at the L1 point between Earth and Sun. $D3^{out}$ is identified by blue squares, $D3^{in}$ by red triangles and D3 by circles. Rows 2 and 5 show linear scaling, allowing $\epsilon^{out/in}$ to be measured. Each curve is 113 values at 64n seconds lag, where $1 \leq n \leq 113$. Each value is the average of estimates from 1134 intervals of 2 days each. Top row: Second moments $S2^{out/in}(\tau)$ of fluctuations in $Z^{out/in}$ and their power spectral density PSD(f) show Kolmogorov scaling: $\tau^{2/3}$, $f^{-5/3}$. (From MacBride, et al. 2008.)

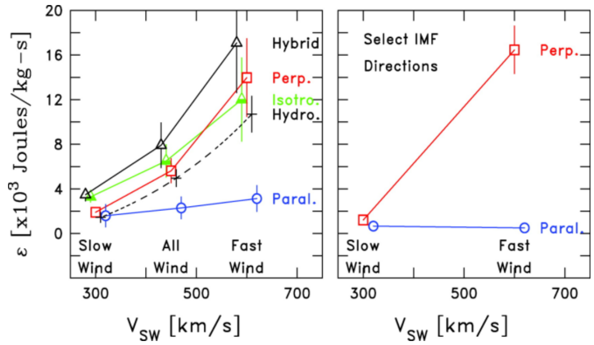

FIGURE 2. Same data as figure1, but averages made for intervals in different types of solar wind, and the anisotropic contributions are called out. (From MacBride, et al. 2008)

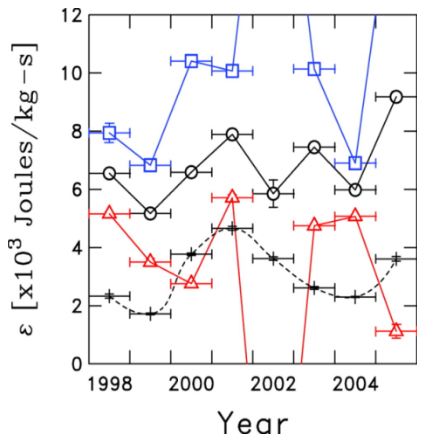

FIGURE 3. Same data as figure 1, but averages made by year. Curves with blue squares are for outward-propagating fluctuations, red triangles for inward-propagating, and black circles is their average, the *energy* dissipation rate. Dashed line is for radial velocity fluctuations alone. (From MacBride, et al. 2008)

2. Ulysses data out of the Ecliptic

Ulysses data is especially interesting for third moments because of the long intervals at solar minimum in wind of high but fairly constant wind speed and unipolar magnetic field which dominate most of the heliosphere. However, Ulysses presents a special challenge for calculating reliable third moments, because of its low and variable plasma data cadence (4 or about 8 minutes, sometimes longer) and 1 hour cadence of the available merged MAG and plasma data set, while the upper scale of the inertial range is only about 1 hour. It is also incorrect to infer in-situ *heating* rates from moments other than D3, as defined in equation (2). These limitations were ignored in the study by Sorriso-Valvo et al. (2007).

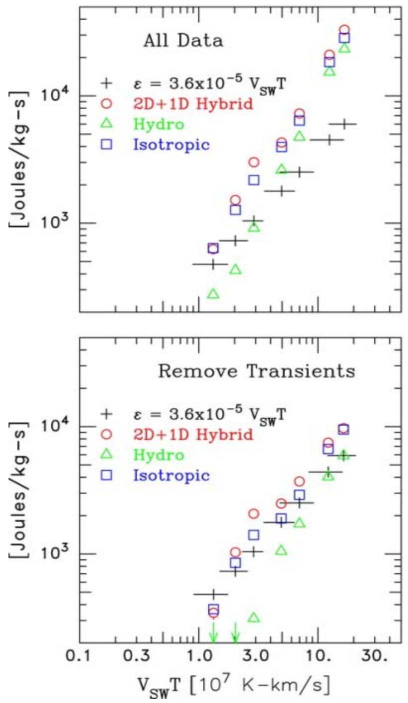

FIGURE 4. ACE merged data (MAG + SWEPAM) 1998-2007. Averages made in ranges of the "VT" of each 12-hour interval. Black crosses are values of proton heating rate inferred from observed non-adiabatic variation of proton temperature with radial distance. (From Stawarz et al. 2009)

D3 (see Equation 2) and its error bars can be calculated by the methods we developed, but at lags longer than a hour or so, no legitimate inference can be made from D3 to the energy cascade rate because the theory does not apply to scales outside the inertial range. Figure 5 shows third moments calculated using an 8-minute data set merged by CWS and JES from available Ulysses data.

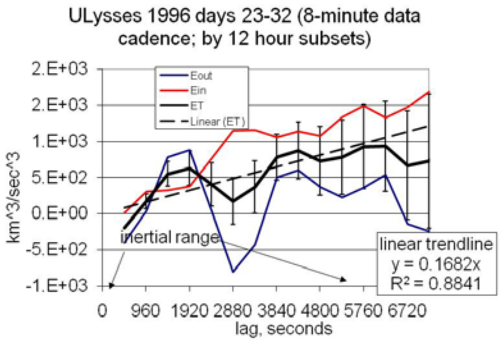

FIGURE 5. Third moments of Z^+ and Z^- fluctuations in Ulysses data, and linear fit to their average.

The slope in Figure 5, with wind speed 750 km/s, yields $\varepsilon = 170 \pm 80$ $\mathrm{Jkg^{-1}s^{-1}}$. This preliminary result is much smaller than any we have measured in the ecliptic and bears further study.

CONCLUSIONS

The exact laws for third-order moments in incompressible MHD turbulence in the inertial range have been used to measure the energy cascade rate in solar wind turbulence in various types of solar wind. This work has established the existence of an anisotropic turbulent energy cascade in the solar wind that is sufficient to account for the observed heating of solar wind plasma in the ecliptic from 0.3 to 1 AU. Statistical techniques to estimate the uncertainties in measuring third moments were developed and used.

ACKNOWLEDGMENTS

The authors thank the *ACE*/SWEPAM team for providing the thermal proton data used in this study. This work was supported by Caltech subcontract 44A-1062037 to the University of New Hampshire in support of the *ACE*/MAG instrument, by NASA Sun-Earth Connection Guest Investigator grants NNX08AJ19G and NNX09AG28G, and NSF SHINE grant ATM0850705.

REFERENCES

B.T. MacBride, C.W. Smith, and M.A. Forman, *Proceedings of the Conference Solar Wind11-SOHO 16, 2005,* edited by B. Fleck and T.H. Zurbuchen, ESA SP-592, ESA Publications Division, Postbus 299, 2200AG Noordwijk, The Netherlands, 2005, pp. 613-616.

B.T. MacBride, C.W. Smith and M.A. Forman, *Astrophys. J.* **679:**1644-1660, 2008

J.J. Podesta, M.A. Forman, C.W. Smith, *Phys. Plasmas*, **14**, 092305, 2007

J.J. Podesta, M.A. Forman, C.W. Smith, D.C. Elton, Y. Malecot, and Y. Gagne, *Nonlin. Proc. Geophys.* **16**, 99-110, 2009

H. Politano, and Pouquet, A., *Phys. Rev. E.*, **57**, 21, 1998

J.E. Stawarz, C.W. Smith, B.J. Vasquez, M.A. Forman and B.T. MacBride, *Astrophys. J.* **697**, 1119-1127, 2009

L. Sorriso-Valvo, R. Marino, V. Carbone, A. Noullez, F. Veltri, R. Bruno, B. Bavassano and E. Pietropaolo, *Phys. Rev. Lett.*, **99**, 115, 2007